

A probabilistic population code based on neural samples

Sabyasachi Shivkumar*, Richard D. Lange*, Ankani Chattoraj*, Ralf M. Haefner *equal contribution



Perception is Probabilistic

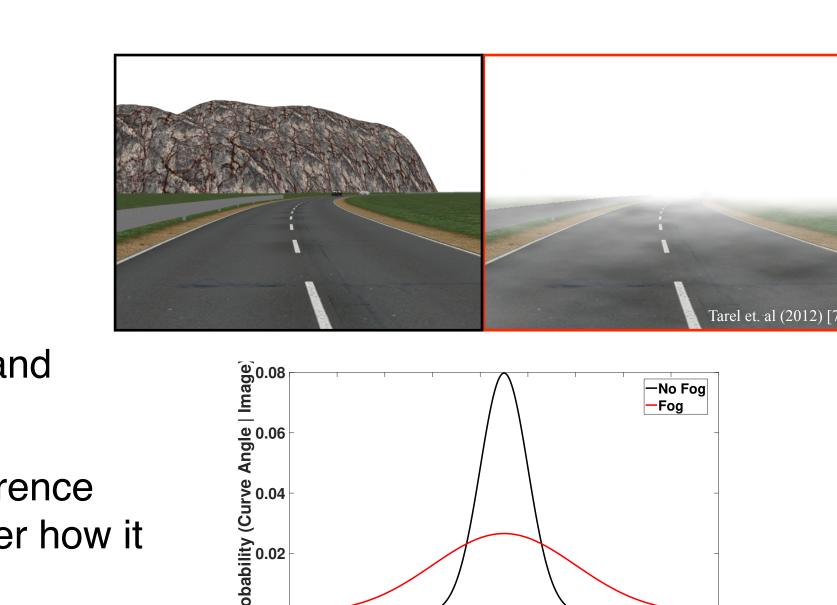
Sensory input is noisy and ambiguous

Inferences made from this input are uncertain

It is behaviorally useful to represent this uncertainty.

Key question: how does the brain represent and compute with probability distributions?

Further Motivation: The brain is the best inference machine we know of; useful to reverse engineer how it works!

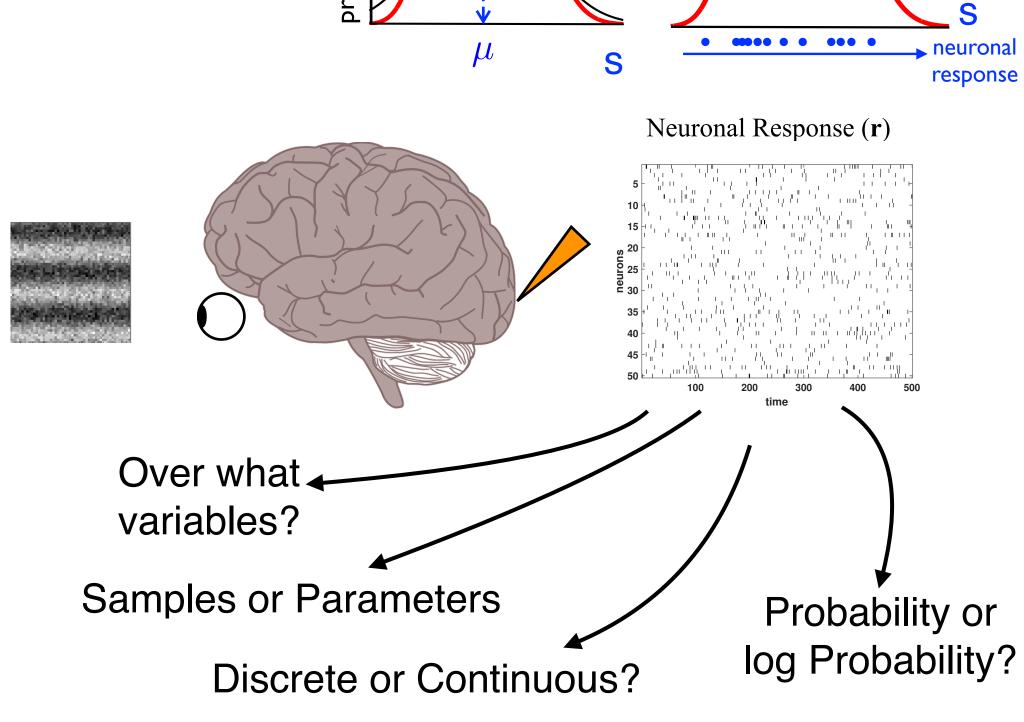


Forward vs Reverse Engineering

Forward Engineering: building a brain by implementing known (approximate) inference algorithms in biologically plausible circuits.

Either assume variational inference or MCMC

Reverse Engineering: observing the brain's inputs, spiking responses, and outputs and interpreting the whole process as inference...



Parametric and Sampling Models

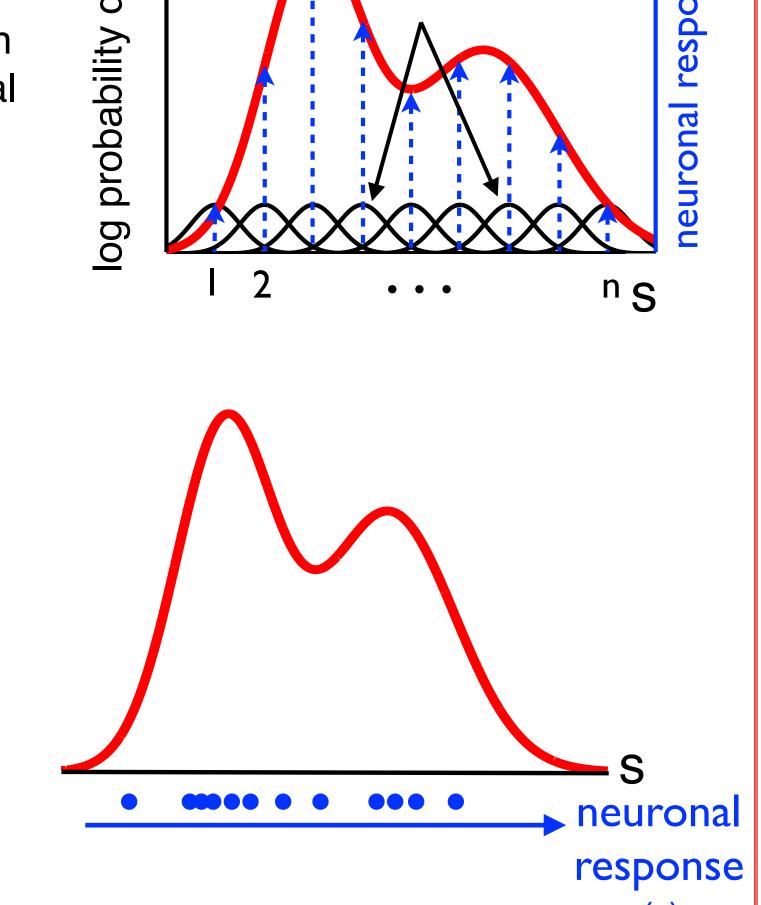
Linear Probabilistic Population Codes (PPCs) [1-2]

- Neuronal population response represent the posterior over orientation (s)
- For a linear PPC, distribution over s belongs to an exponential family with the firing rate (r) as natural parameters
- n neurons : 1 variable

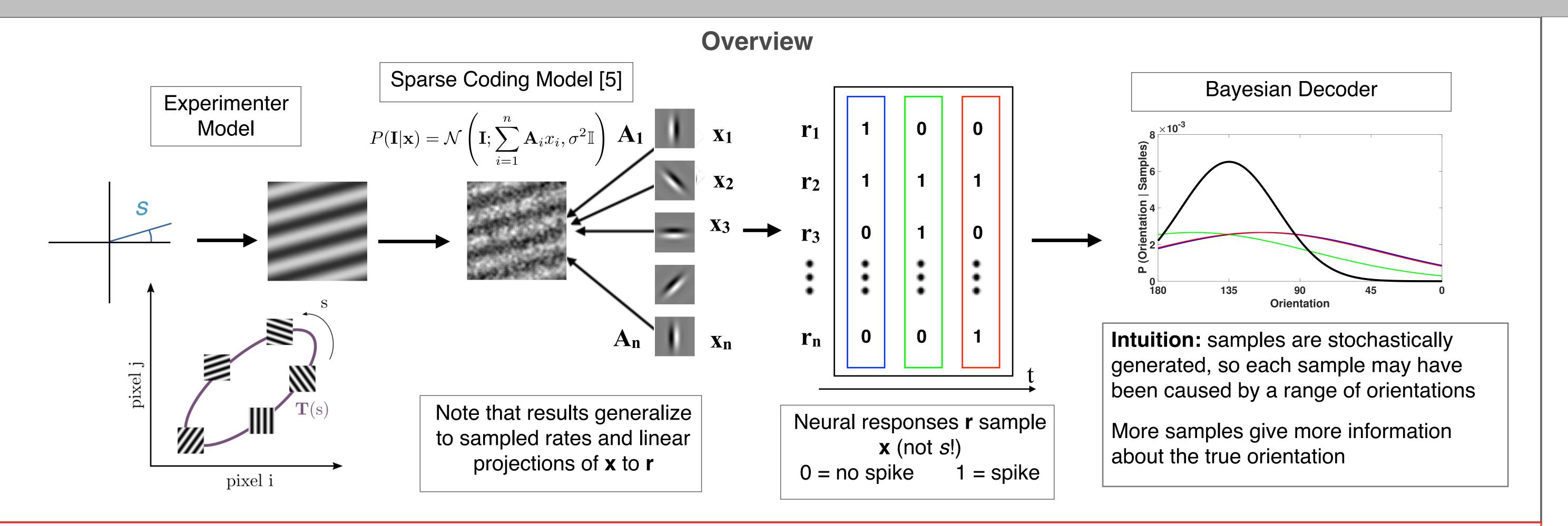
$$P(s|\mathbf{r}) \propto \exp\left(\sum_{i=1}^{n} h_i(s)r_i\right)$$

Neural Sampling Hypothesis [3-4]

- Neural dynamics as MCMC sampling
- Population encodes discrete values at a given time, uncertainty is encoded over time
- n neurons : n variables



h_i(s)



Key result: Bayesian Decoder is a Linear PPC!

Derivation

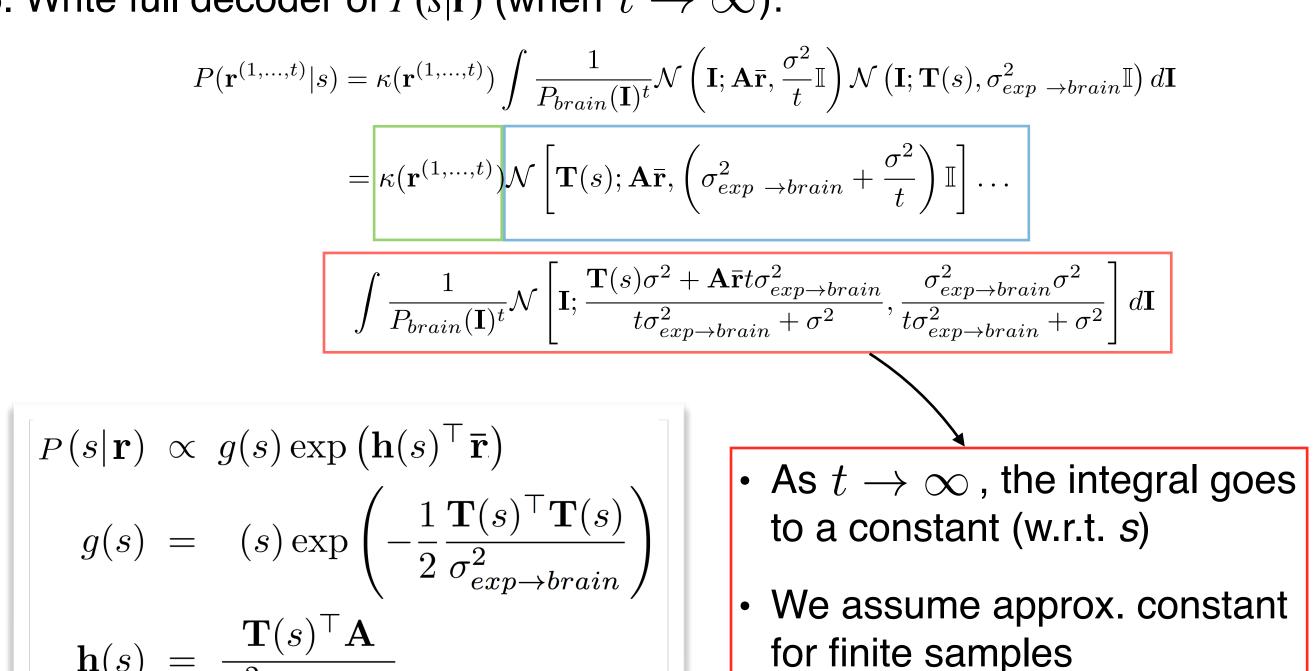
- 2. Define P(rII) using the encoding model

$$P(\mathbf{r}|\mathbf{I}) = \frac{P_{brain}(\mathbf{I}|\mathbf{r})P_{brain}(\mathbf{r})}{P_{brain}(\mathbf{I})}$$

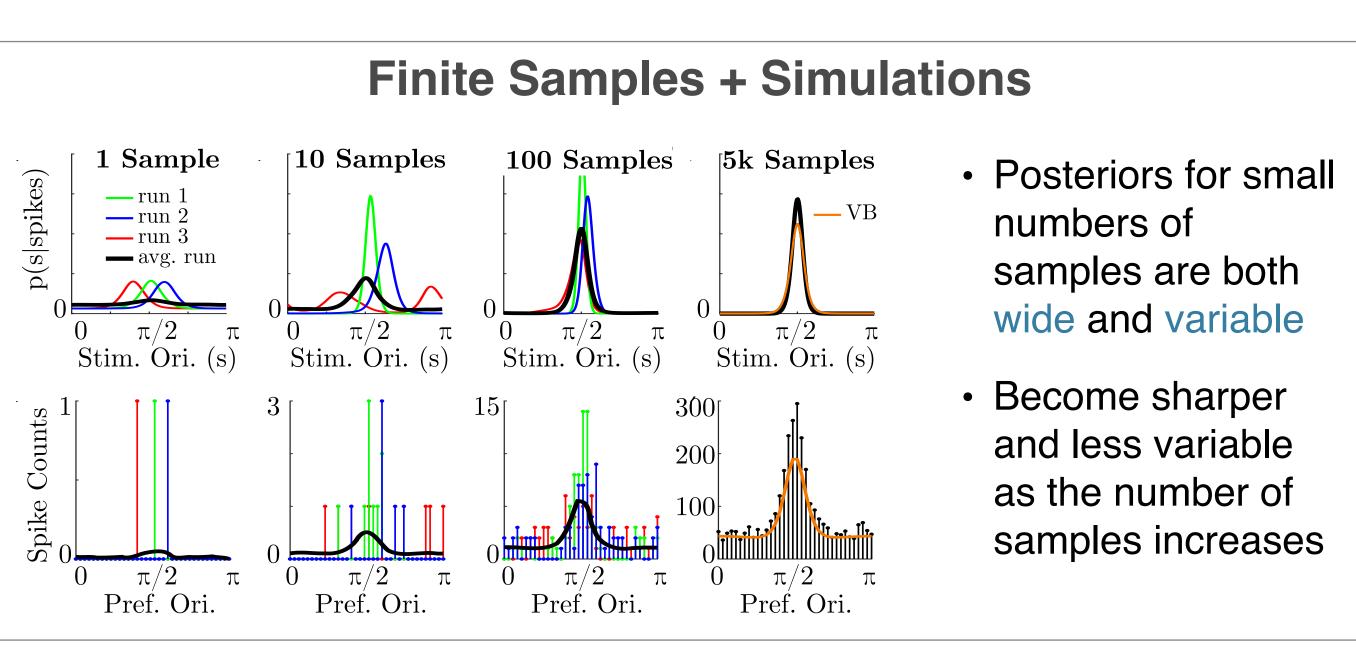
3. Likelihood of s given t samples of t requires marginalizing over all possible images that could have "caused" the given set of samples

$$P(\mathbf{r}^{(1,\dots,t)}|s) = \int d\mathbf{I} P(\mathbf{I}|s) P(\mathbf{r}^{(1,\dots,t)}|\mathbf{I})$$
$$= \int d\mathbf{I} P(\mathbf{I}|s) \prod_{t'=1}^{t} P(\mathbf{r}^{(t')}|\mathbf{I})$$

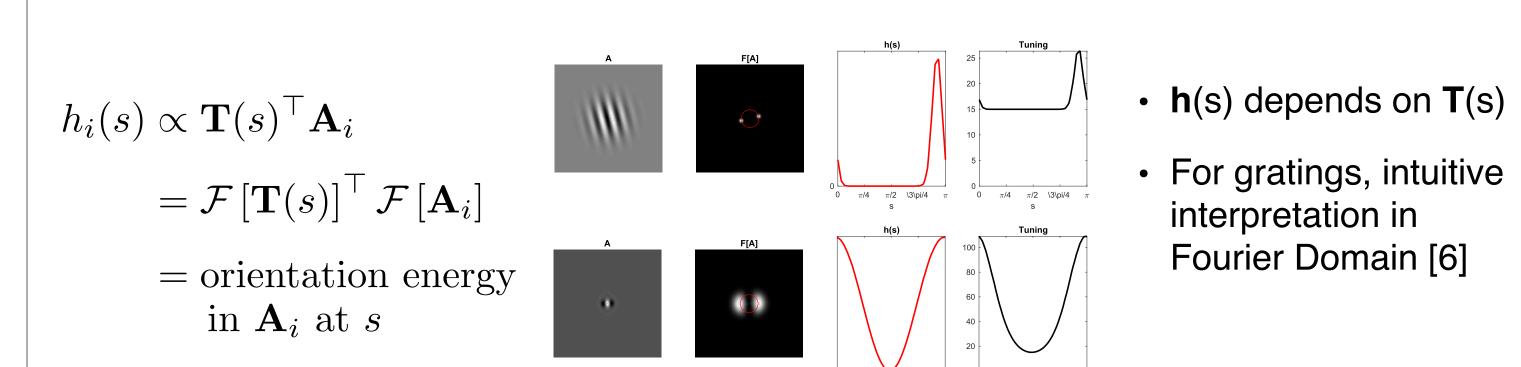
- 4. Recursively expand product over samples, resulting in product of terms:
- i. terms independent of s
- ii. term depending on **mean** of samples
- iii. integral that vanishes (becomes constant) as $t o \infty$
- 5. Write full decoder of $P(s|\mathbf{r})$ (when $t \to \infty$):



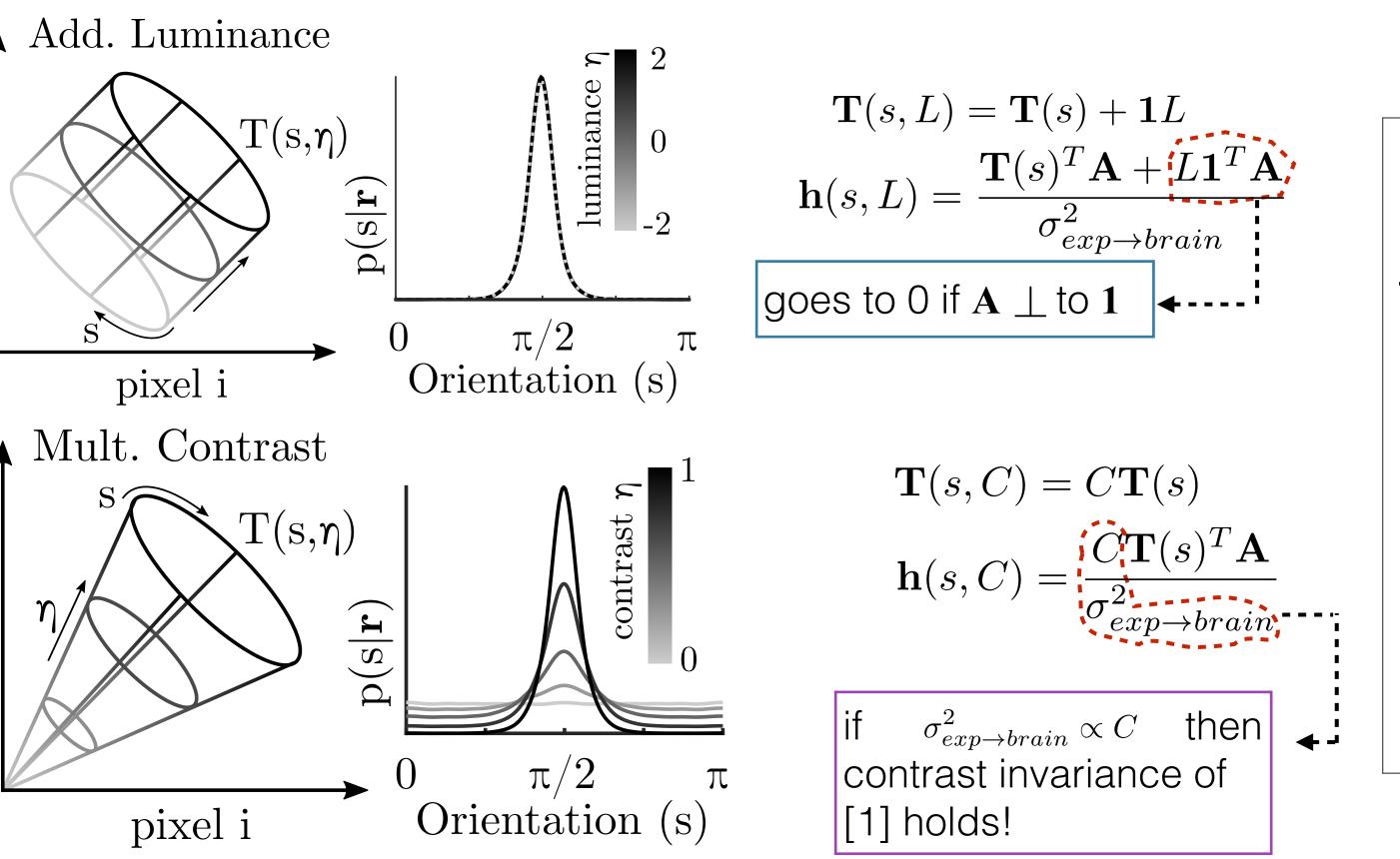
• Likelihood based on distance in image space between reconstruction and manifold points. • Even for gratings, manifold is complicated



Projective Fields and Log Probability Kernels



Nuisance Variables

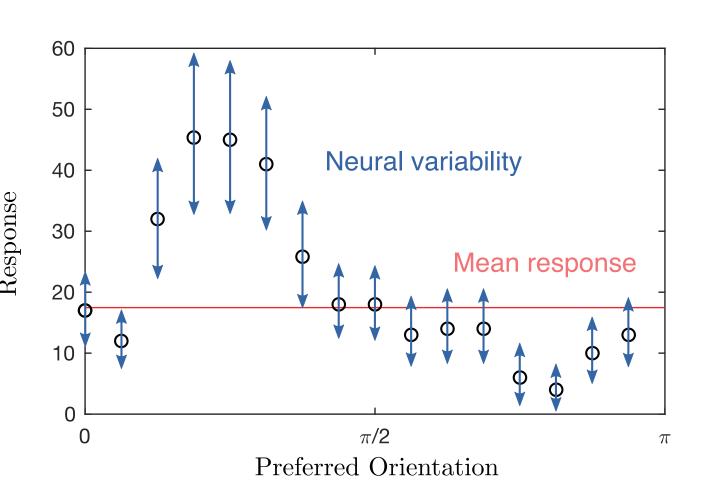


Nuisance variables extend the T(s) manifold to $T(s,\eta)$

Two notions of invariance:

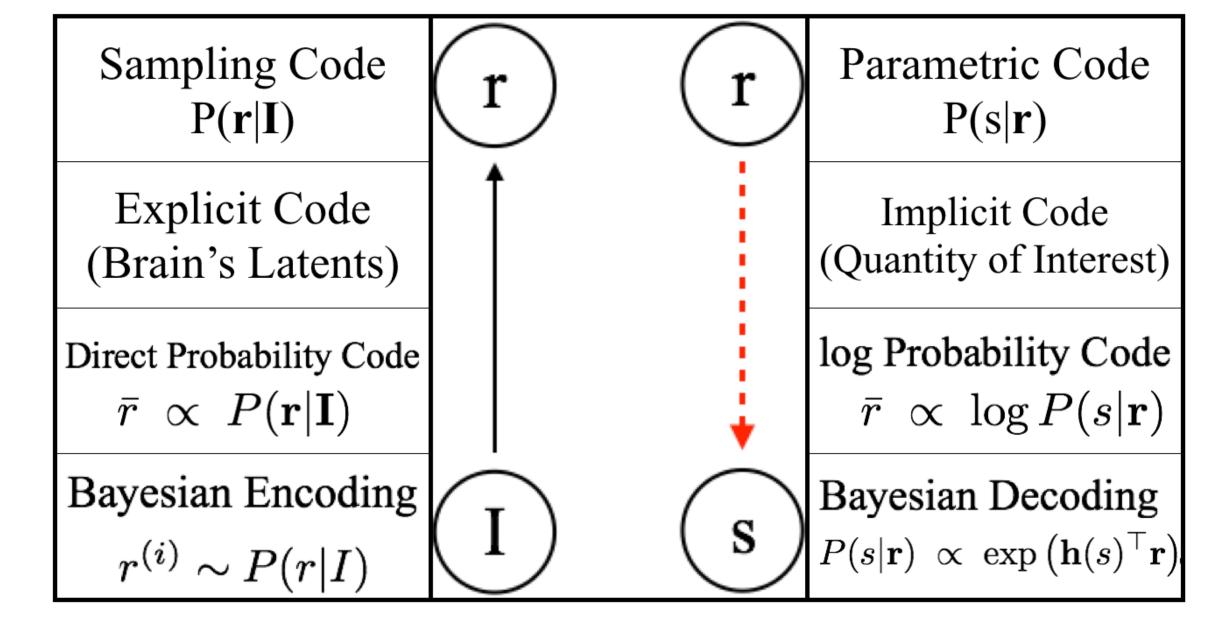
- 1. The posterior distribution is invariant to luminance, but it depends on contrast
- 2. **h** is independent of η (this makes decoding the PPC easier) [1]
- (2) is true for contrast if noise scales with contrast

The Relationship between Uncertainty and Variability



- Intuition: a hallmark of sampling is that wider posteriors result in more variable responses
- Uncertainty over **x** results in more variable responses as expected
- <u>Uncertainty over s</u> is distributed across the population, determined by the population's mean response <u>independent of neural variability</u>

Implications and Summary



- PPC and Sampling are not mutually exclusive representations (if over different variables)
- Analogous to decoding task-specific quantities from a pre-trained model
- Simultaneous representation of different classes of probability codes is possible in the same system
- Results hold for any decoded variable, not just orientation
- Results hold for any internal model learned by the brain as long as decoded image converges to a delta as number of samples increases
- Suggests other possible coding schemes like sampled mixtures-of-PPCs

References

[1] Ma, W. J., Beck, J. M., Latham, P. E., & Pouget, A. (2006). Bayesian inference with probabilistic population codes. Nature Neuroscience.
[2] Beck, J. M., Ma, W. J., Kiani, R., Hanks, T., Churchland, A. K., Roitman, J., ... Pouget, A. (2008). Probabilistic Population Codes for Bayesian Decision Making. Neuron.
[3] Hoyer, P. O., & Hyvärinen, A. (2003). Interpreting neural response variability as monte carlo sampling of the posterior. Advances in Neural Infromation Processing Systems.
[4] Orbán, G., Berkes, P., Fiser, J., & Lengyel, M. (2016). Neural Variability and Sampling-Based Probabilistic Representations in the Visual Cortex. Neuron.
[5] Olshausen, B. a, & Field, D. J. (1997). Sparse coding with an incomplete basis set: a strategy employed by V1? Vision Research.
[6] Freeman, W. T., & Adelson, E. H. (1991). The Design and Use of Steerable Filters. IEEE Trans Pattern Anal Mach Intell.
[7] Tarel, Jean-Philippe, et al. (2012). "Vision enhancement in homogeneous and heterogeneous fog." IEEE Intelligent Transportation Systems Magazine.